

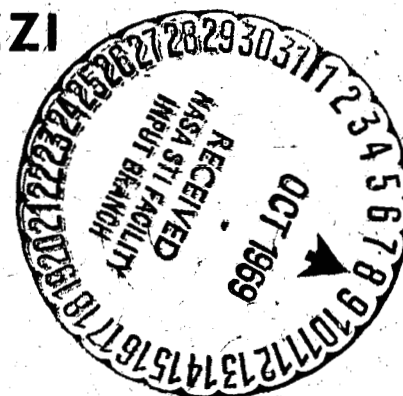
X-734-69-408
PREPRINT

NASA TM X- 63678

THE USE OF SUBSONIC ORIFICES FOR MEASUREMENT OF GAS FLOW

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SEPTEMBER 1969



GSFC

GODDARD SPACE FLIGHT CENTER
GREENBELT, MARYLAND

FACILITY FORM 602

N 69-37579

(THRU)

(PAGES)

(CODE)

NASA-TM X-63678

(NASA CR OR TMX OR AD NUMBER)

(CATEGORY)

X-734-69-408

THE USE OF SUBSONIC ORIFICES FOR
MEASUREMENT OF GAS FLOW

by

Thomas A. Cygnarowicz and Ronald Corazzi
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ABSTRACT

An approximate equation is derived for calculation of gas weight flow rate through subsonic orifices for small differential pressures. Comparison to the exact equation shows good correlation even at conditions far removed from those assumed in the derivation. A limited test program verifies the applicability of the approximation.

SYMBOLS

A_{12} = Orifice cross section area
 P_1 = Pressure upstream of orifice
 P_2 = Pressure downstream of orifice (thruster inlet pressure)
 ΔP = $P_1 - P_2$
 T_1 = Temperature upstream of orifice
 g = Acceleration of gravity
 C_D = Coefficient of discharge
 R = Gas constant
 \dot{w} = Exact flow rate of gas
 \dot{w}_a = Approximation for \dot{w} given by eq. (7)
 \dot{w}_m = Measured flow rate of gas
 γ = Specific heat ratio

INTRODUCTION

The use of sonic orifices in the measurement of gas flow rates between 10^{-5} and 10^{-4} lbs/sec for laboratory performance evaluation of millipound thrust ammonia resistance jet thrusters has been common practice (Ref.'s (1) and (2)). This type of testing, however, usually does not fully simulate the operation of the thruster on a spacecraft. The prime requirement on a spacecraft is for a constant thrust which dictates that the inlet pressure to the thruster be held constant. For a long gas pulse, the specific impulse (thrust/ \dot{w}) of an ammonia resistance jet could vary by a factor of two from initiation to completion of the pulse due to the decrease in gas temperature at the thruster nozzle as the temperature of the resistance jet drops. This means that if the thrust is held constant, the flow rate at the completion of the pulse would be double the initial flow rate. If a sonic measuring orifice is placed in front of the thruster, the requirement of a constant downstream pressure (P_2) is very difficult to maintain because P_1 has to be continuously varied as the flow rate increases. With a subsonic orifice, however, P_2 can be held essentially constant with only small variations of P_1 .

APPROXIMATION OF ORIFICE FLOW EQUATION

The exact expression for the weight flow of a compressible fluid through an orifice is given by

$$\dot{w} = \frac{A_{12} P_1}{\sqrt{T_1}} C_D \left\{ \frac{2\gamma}{(\gamma-1)R} \left[\left(\frac{P_2}{P_1}\right)^{2/\gamma} - \left(\frac{P_2}{P_1}\right)^{\frac{\gamma+1}{\gamma}} \right] \right\}^{0.5} \quad \text{eq. (1)}$$

By using a series expansion for the pressure ratio terms under the radical in eq. (1), Ref. (3) derives an approximation for the orifice flow equation when ΔP is small:

$$\left(\frac{P_2}{P_1}\right)^{2/\gamma} = 1 - \frac{2}{\gamma} \frac{\Delta P}{P_1} + \frac{2-\gamma}{\gamma^2} \left(\frac{\Delta P}{P_1}\right)^2 - \dots \quad \text{eq. (2)}$$

$$\left(\frac{P_2}{P_1}\right)^{\frac{\gamma+1}{\gamma}} = 1 - \frac{\gamma+1}{\gamma} \frac{\Delta P}{P_1} + \frac{\gamma+1}{2\gamma^2} \left(\frac{\Delta P}{P_1}\right)^2 - \dots \quad \text{eq. (3)}$$

By neglecting terms in $(\frac{\Delta P}{P_1})$ to the third power or greater, the following equation is obtained:

$$\left(\frac{P_2}{P_1}\right)^{2/\gamma} - \left(\frac{P_2}{P_1}\right)^{\frac{\gamma+1}{\gamma}} = \frac{\gamma-1}{\gamma} \frac{\Delta P}{P_1} \left(1 - \frac{3}{2\gamma} \frac{\Delta P}{P_1}\right) \quad \text{eq.(4)}$$

Substitution of eq.(4) into eq.(1) yields

$$\dot{W} \approx A_{12} C_D \left\{ \frac{2g P_1}{RT_1} \left(1 - \frac{3}{2\gamma} \frac{\Delta P}{P_1}\right) \Delta P \right\}^{0.5} \quad \text{eq.(5)}$$

Since eq.(5) is valid only for small ΔP 's, it is reasonable to assume that the term $(3/2\gamma)(\Delta P/P_1)$ should also be small. Compressible gases of interest have specific heat ratios of 1.0 to 1.67. It is reasonable to further assume, therefore, that setting $(3/2\gamma) \approx 1$ would introduce negligible error into the flow rate approximation. Eq.(5) thus becomes

$$\dot{W}_a = A_{12} C_D \left[-\frac{2g}{RT_1} \right]^{0.5} [P_2 \Delta P]^{0.5} \quad \text{eq.(6)}$$

For the application of interest, the gas temperature at the orifice is essentially constant at ambient conditions. If one further assumes that $C_D = \text{constant}$ for a particular orifice, eq.(6) can be written

$$\dot{W}_a = K_g (P_2 \Delta P)^{0.5} \quad \text{eq.(7)}$$

where

$$K_g = A_{12} C_D \left(-\frac{2g}{RT_1} \right)^{0.5} \quad \text{eq.(8)}$$

Thus with ΔP small, $\gamma \approx 1.5$, $C_D = \text{constant}$ and $T_1 = \text{constant}$, the weight flow varies linearly with $(P_2 \Delta P)^{0.5}$.

To test the validity of the assumptions used in deriving eq.(7) calculations were made on a digital computer comparing the weight flow calculated by eq.(7) to that calculated by eq.(1). Various gammas were used and the pressure ratio was varied from $P_2/P_1 = 1.0$ down to the critical pressure ratio. Percentage error was found from

$$\% \text{ Error} = \frac{|\dot{W} - \dot{W}_a|}{\dot{W}} \times 100 \quad \text{eq.(9)}$$

It can be shown that percentage error is independent of A_{12} , R , C_D , g and T_1 , and can be expressed as the following function of (P_2/P_1) and γ :

$$\% \text{ Error} = \left\{ 1 - \frac{\left(1 - \frac{P_2}{P_1}\right) \left(\frac{\gamma-1}{\gamma}\right)}{\left(\frac{P_2}{P_1}\right)^{2-\frac{1}{\gamma}} - \left(\frac{P_2}{P_1}\right)^{1/\gamma}} \right\}^{0.5} \times 100 \quad \text{eq. (10)}$$

Table 1 summarizes the data found from this analysis.

TABLE 1: $\% \text{ Error} = \frac{|\dot{w} - \dot{w}_a|}{\dot{w}} \times 100$

$P_2/P_1 \backslash \gamma$	1.14	1.31	1.4	1.5	1.66
.98	.32	.15	.07	.0008	.10
.95	.83	.38	.19	.005	.24
.90	1.72	.79	.40	.02	.49
.85	2.68	1.24	.63	.005	.73
.80	3.73	1.73	.90	.09	.98
.75	4.88	2.27	1.19	.15	1.23
.70	6.14	2.88	1.53	.24	1.48
.65	7.53	3.55	1.91	.34	1.74
.60	9.08	4.31	2.35	.48	1.99
$(P_2/P_1)_{\text{crit}} = .577$	9.85				
$(P_2/P_1)_{\text{crit}} = .544$		5.28			
$(P_2/P_1)_{\text{crit}} = .528$			3.10		
$(P_2/P_1)_{\text{crit}} = .512$.83	
$(P_2/P_1)_{\text{crit}} = .488$					2.53

Additional calculations show that using eq.(5) introduces less error than the use of eq. (7). This is to be expected because eq.(5) uses the correct value of γ . Eq.(7), however, expresses \dot{w} as a linear function of $(P_2 - P_1)^{0.5}$, and is more convenient when calibrating an orifice.

The data of Table 1 shows that the approximate equation for \dot{w} gives good correlation with the exact equation even at conditions far removed from those assumed in deriving eq.(7). As expected the amount of error decreases as ΔP and γ approach zero and 1.5 respectively. Table 1 shows, however, that even at critical flow, the error introduced by using eq.(7) is less than 10% for a gas with $\gamma = 1.37$ and approximately 2.5% for a gas with $\gamma = 1.66$. Eq.(7) gives a high estimate of the flow rate for the four gammas considered with $\gamma \leq 1.5$ and a low estimate for the $\gamma = 1.66$ gas.

The principal value of eq.(7) and the limited supporting orifice calibration data, besides providing a simpler equation than that of eq.(1) for estimating the flow rate through an orifice, is the indication that only a few points need be taken in the calibration of the orifice to

obtain the straight line relationship of \dot{W} vs. $(P_2 \Delta P)^{0.5}$.

An interesting observation is that a linear relationship between \dot{W} and $(P_1 \Delta P)^{0.5}$ can be obtained by assuming that $(3/2\gamma)(\Delta P/P_1)$ is negligible with respect to unity in eq.(5). This approximation, however, is less accurate (in comparison to the exact expression for \dot{W}) than eq. (7) at all pressure ratios for gases with $\gamma \leq 1.5$. For gases with $\gamma > 1.5$, this approximation gives more accurate results for very small ΔP 's, but it gives significantly larger errors as the orifice pressure ratio approaches critical conditions (e.g. at $\gamma = 5/3$, and $P_2/P_1 = .488$, this approximation gives 40% error).

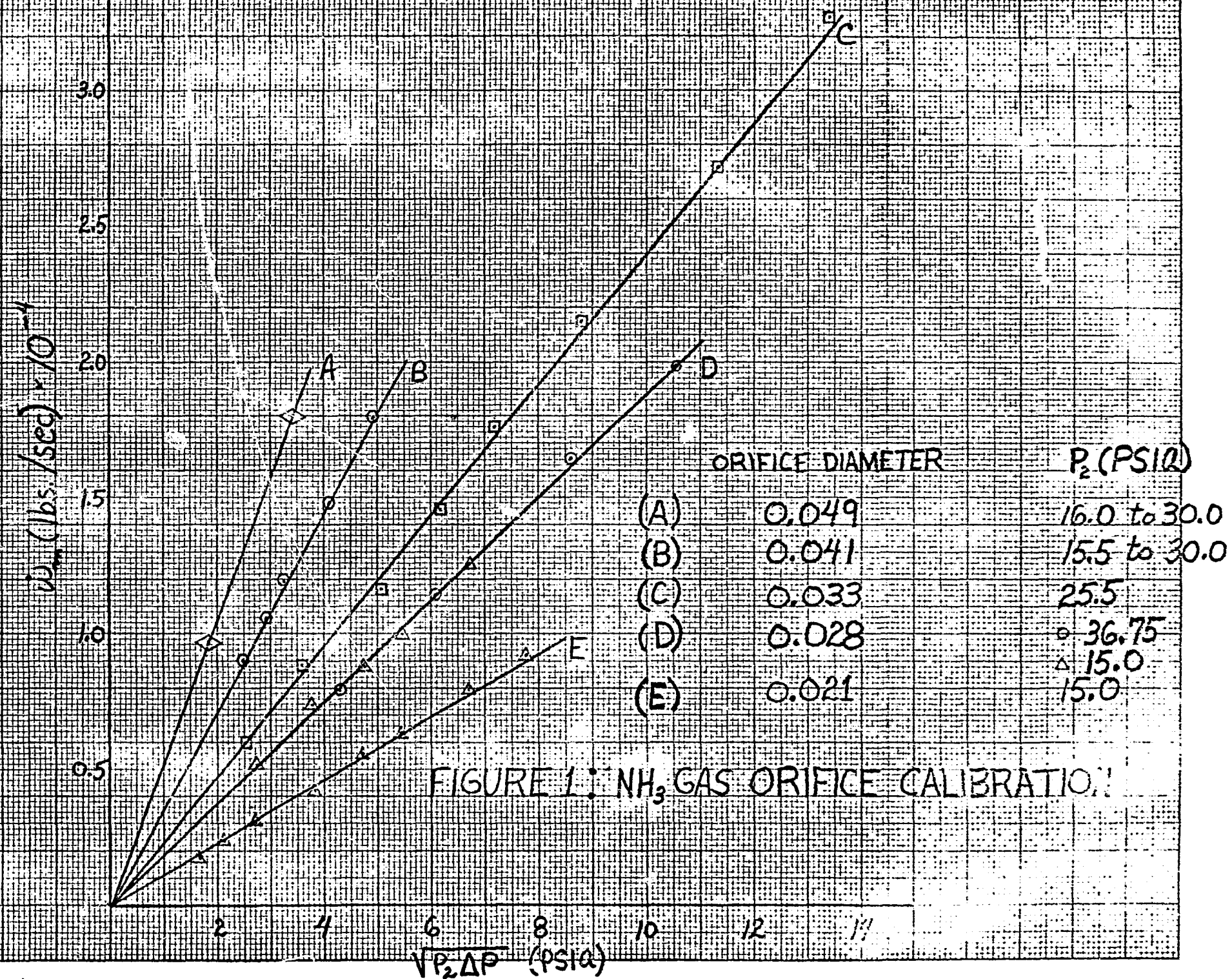
ORIFICE CALIBRATION

To further verify that \dot{W} can be approximated as a linear function of $(P_2 \Delta P)^{0.5}$, a number of small diameter orifices were calibrated with NH_3 . The calibration procedure is detailed in the Appendix. The results of the calibration, shown in Figure 1, verify the linear relationship between \dot{W} and $(P_2 \Delta P)^{0.5}$. The values obtained for the discharge coefficients for these orifices, which were made by drilling holes in a sheet of .0625 inch thick stainless steel, varied from .77 to .94 (assuming the orifice diameter measurements were correct) but appeared to be constant for each particular orifice in the range tested.

The orifice calibrations were limited to cases where the pressure ratios (P_2/P_1) were large (the calibration was done to satisfy the flow requirements of a resistance jet test program). More extensive testing would be required to verify that C_D , for a particular orifice, is constant over a larger pressure ratio variation.

REFERENCES

1. Cygnarowicz, T.A., and Gibson, R. N., Jr., "Design and Performance of a Thermal Storage Resistojet", Paper 67-662, 1967 AIAA; also Journal of Spacecraft and Rockets, Vol. 5, No. 6, June, 1968, pp. 686-689.
2. Viventi, R. E., et al., "Simulated Radioisotope Thrust System and Component Development", Contr. NAS 5-9670, General Electric Co. Report GESP-87, no date.
3. Andersen, B. W., The Analysis and Design of Pneumatic Systems, John Wiley and Sons Inc., New York, 1967, pp. 17-22.



APPENDIX

ORIFICE CALIBRATION PROCEDURE

Figure A shows the schematic of the orifice calibration system. The orifice is calibrated by the following procedure:

- (1) Fill the sphere with gas.
- (2) Weigh the filled sphere (use 10,000 gram capacity analytic balance with a ± 0.1 gram readability).
- (3) Attach the sphere to the flow system as shown in Figure A. The pressure regulator and valve (B) are preset by trial runs to yield the desired P_2 and ΔP .
- (4) Open valve (A) and start a stop watch at the same time. Let the gas flow for a sufficient amount of time (depending on orifice size) to offset transient and measurement errors.
- (5) Close valve (A) and turn off stop watch at same time.
- (6) Remove the sphere from the flow system and weigh it.
- (7) The flow rate is found by subtracting the final weight of the sphere from the initial weight and dividing by the time period for which the gas was flowing.
- (8) Vary P_2 , and ΔP over a large enough range to plot the curve of \dot{w} vs $(P_2 \Delta P)^{0.5}$ (should be a straight line for small ΔP 's) for the range of interest.

The calibration system shown in Figure A exhausts the gas leaving valve (B) at ambient pressure. This valve can be replaced by an orifice which exhausts to a vacuum (to simulate the thruster). Some orifices were calibrated by both techniques and gave identical calibration curves.

The water bath shown in Figure A is used to maintain the test gas at a uniform temperature by means of a temperature controller.

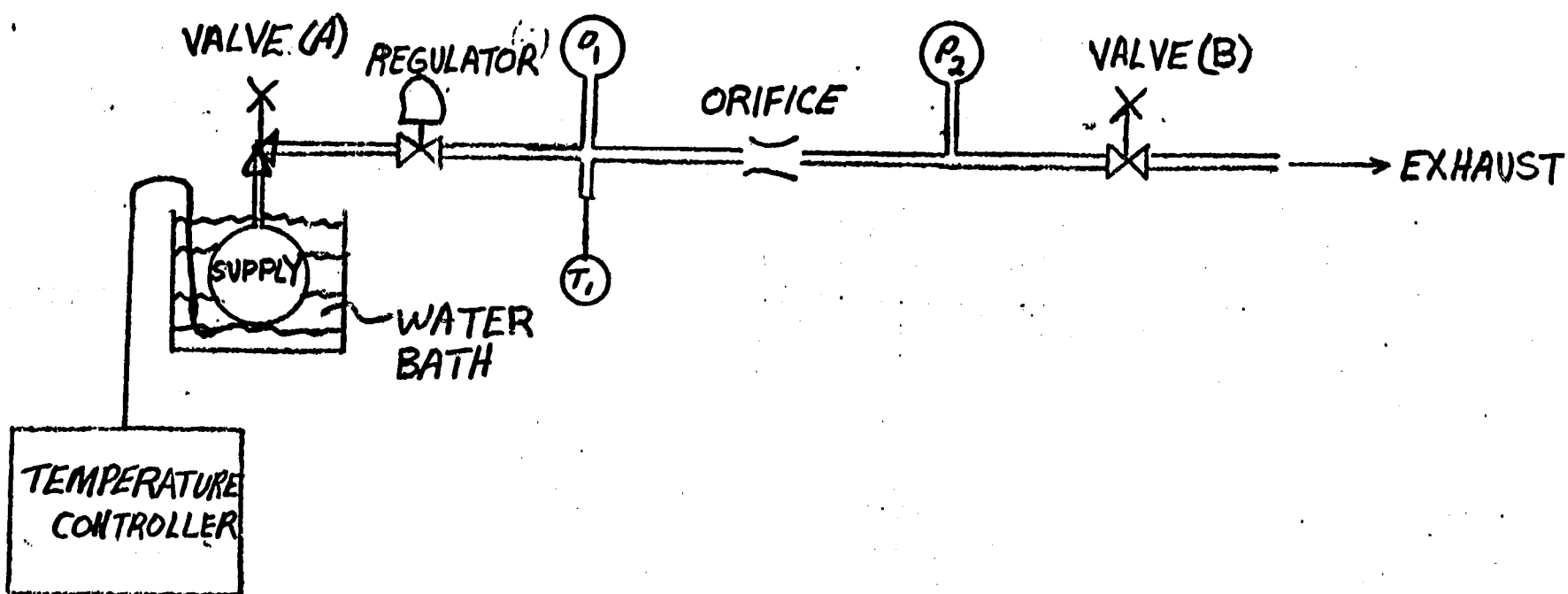


FIGURE A: ORIFICE CALIBRATION SYSTEM